Metabolism - Age of Carbon Working Group: Modeling Effort #1

In this work, we develop a simple model for the concentration and age of carbon in a lake. The lake receives carbon loads from (1) surface runoff as stream flow, (2) groundwater seepage, and (3) direct precipitation, as follows:

$$V_L \frac{dC_L}{dt} = Q_S C_S + Q_G C_G + Q_P C_P - Q_{Lo} C_L - \lambda_L C_L V_L \qquad (1)$$

where Q is flow [L^3/T], C is DOC concentration [M/L^3], V is volume [L^3] and λ is the DOC decay rate constant [T^{-1}]. The subscripts signify stream (*S*), groundwater (*G*), lake (*L*), evaporation (*E*), and (*Lo*) signifies the lake outflow. Flow conservation dictates that

$$\frac{dV_L}{dt} = Q_S + Q_G + Q_P - Q_E - Q_{Lo} \qquad (2)$$

Initially, we might assume that the stream and groundwater behave as "plug flow" reactors, which enables us to define their output concentrations (which is the lake input concentration) independently according to (3a) and (3b). To define these, we would need to use literature/data on DOC concentrations and transformation rates and site-specific geometry and hydraulic considerations.

$$C_G = C_{Go} e^{-\lambda_G \tau_G} \qquad (3a)$$

$$C_S = C_{So} e^{-\lambda_S \tau_S} \qquad (3b)$$

where C_{Go} and C_{So} are the headwater or upstream groundwater and stream DOC concentrations, respectively, and τ is the residence time for the respective system. For groundwater, the residence time is:

$$\tau_G = \frac{L_G \phi}{q_G} = \frac{L_G \phi}{K_H \left(\frac{dh}{dl}\right)} \qquad (4a)$$

$$\tau_S = \frac{L_S}{U_S} \qquad (4b)$$

where L_G is the average groundwater flow path distance, ϕ is the sediment porosity, and q_G is the groundwater's average Darcy velocity (specific discharge), which estimated from the groundwater hydraulic gradient (*dh/dl*) and the hydraulic conductivity (K_H) of the aquifer sediments. For the stream, L_S is the stream length and U_S is the average velocity.

The precipitation term in (2) should be handled as periodic terms of time-variable flow based on preciptiation rate (*P*) and lake area:

$$\begin{aligned} Q_P &= PA_L \ during \ event \\ Q_P &= 0 \ otherwise \end{aligned} \qquad (5)$$